AMC 8 and MATHCOUNTS Geometry Day 1

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0.1 Lesson Overview

In this lesson we will learn about the concept of area and how to use it to solve problems from AMC 8, MATHCOUNTS and similar contests. Our strategy is to start with motivating problems and then use these problems to learn underlying concepts. The core concepts that we will explore are the following:

1. Side angle side rule of congruency
2. Carpet Strategy

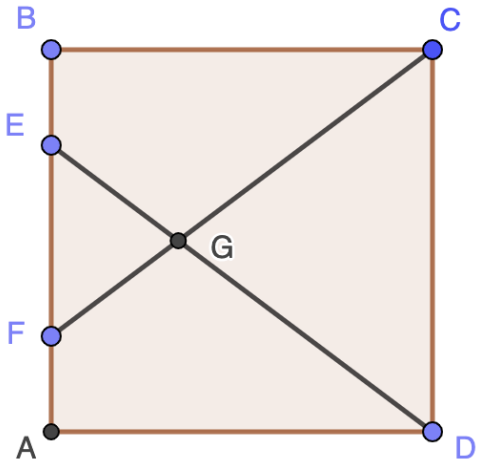
0.2 Intuition of Area

Intuitively speaking, area of an object is the measure of space covered in the plane by that object. In ancient times architect and civil engineers used geometric tools to measure area. They needed to know this data precisely to make constructions.

To understand how we can measure area, we begin with a motivating problem. As we will see, **no formula is necessary to solve this problem.**

0.3 Problem 1

Suppose $ABCD$ is a square of side length 4. Let E and F be points on AB such that $AF = BE = 1$. Show that quadrilateral $CBEG$ has area 4.67 then what is the area of $AFGD$?



Let us learn a few concepts to understand this problem.

0.3.1 Congruency

Two geometric figures are know as congruent if, informally speaking, you can place one figure exactly on the top of another and they sizes and shapes exactly fit with each other.

In particular if two figures are congruent then they have equal areas.

There are several ways to determine if two triangles are congruent or not. One important way is the following rule.

Side-Angle-Side rule.

Suppose ΔABC and ΔXYZ be two triangles such that two sides of ΔABC are equal in length with two sides of ΔXYZ. Moreover suppose the included angles are also equal. Then we say that the triangles are congruent by side angle side rule.

Challenge: Can you show that the above rule works. That is if side-angle-side of one triangle is equal to side-angle-side of another triangle then you can actually place one triangle exactly on the top of another.

0.3.2 Carpet Strategy

Carpet Strategy is useful to compare area covered by two different figures. The strategy is best illustrated by example. In the above figure, notice that ΔBCF and ΔADE has two sides of equal lengths.

$$BC = AD = 4$$

$$BF = AE = 3$$

Also notice that the included angle of the two equal sides are equal.

$$\angle CBF = \angle DAE = 90^o$$

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Hence the two triangles ΔBCF and ΔADE are congruent meaning one of them if placed on another will exactly fit in. Therefore these two triangles have equal area. Now notice that ΔBCF and ΔADE overlap each other in the region ΔEGF. Hence if we remove the overlap region then left out portions will have equal area.

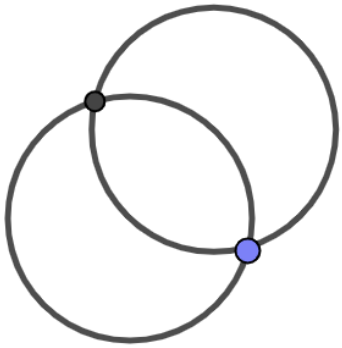
$$\Delta BCF - \Delta EGF = \Delta DAE - \Delta EGF$$

This shows that quadrilateral $CBEG$ and $AFGD$ has the same area.

0.4 Try yourself

0.4.1 Problem 1

Two circles of same radius are overlapping each other. Can you shade two distinct regions with same area in this picture?



0.4.2 Problem 2

Source: Cut-the-Knot

If the blue colored region as area 23 what is the area of the red colored region?