Winter Bose Math Olympiad - Intermediate Main

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1 Problems

1. Let t be a natural number. Suppose the ten's place digit of t^2 is odd. Then the unit's place digit is

2. There are two trees A and B on a field such that the distance between A and B is 5 meters. Ayesha is continuously running on the field such that sum of her distances from A and B is always 5 meters. How many times does she visit the midpoint of A and B?

3. Suppose you have infinitely many carpets, each 73 meters long, 30 meters wide. There are big fields in your school, each 28 meters long, 30 meters wide. Each field is immediately adjacent to the other along the length. After covering t fields with carpets, you found that exactly 1 meter is left uncarpeted. What is the smallest value of t?

4. Suppose there is a 3 cross 3 board and you have two colors: red, blue. How many ways can you color the board?

5. Suppose the ancient Atlantis language has 4 letters , , , . They used these letters to make words of at most 3 letters. How many words are there in the ancient Atlantis language?

6. In ancient Bengal, there were 2020 cities, each connected with another by one or more roads. It is known that there are 4 roads coming out of each city. How many intercity roads were there in ancient Bengal? (The intercity road is a road that connects two cities and has no other city in between).

7. In wonderland lives 3 squares, 4 triangles, and 5 circles. Whenever two different shapes meet, they both magically transform into the third type of shape. For example, when a square and a triangle meet, the square becomes a circle and the triangle becomes the circle.

After 2020th such transformation, what is the remainder if (number circles−number of triangles) is divided by 3?

8. Five friends Amal, Bimal, Kamal, Parimal, and Sreyan want to make a boat. Amal donates 4 sticks of wood, Bimal donates 5 sticks of wood, Kamal donates 5 sticks of wood and Parimal donates 6 sticks of wood. Sreyan says, 'I have no wood stick. I will give 20 gold coins. Please divide it amongst four of you.' How many gold coins should Parimal receive?

   that $(x-y)(y-z)(z+x) = -90$ and $(x-y)(y+z)(z-x) = 42$. Find the value of $z(x-y)^2$.

9. Find the smallest positive multiple of 35 whose digits are all the same as each other.

2 Your Problem Solving Strategies

- Try small cases. Plug in small numbers. Do examples. Consider extreme cases.
- Modify the problem. Generalize and De-Generalize. Look for patterns.
- Draw pictures. Look for symmetry.
- Divide into cases. Work backwards.
- Choose effective notation. Don't be afraid of a little algebra.
- Ask Questions. Argue by contradiction.
- Don't give up after five minutes.