1. You are given a 4×4 chessboard, and asked to fill it with five 3×1 pieces and one 1×1 piece. Then, over all such fillings, the number of squares that can be occupied by the 1×1 piece is

(A) 4 (B) 8 (C) 12 (D) 16.

2. A brand called Jogger's Pride produces pairs of shoes in three different units that are named U_1, U_2 and U_3. These units produce $10\%, 30\%, 60\%$ of the total output of the brand with the chance that a pair of shoes being defective is $20\%, 40\%, 10\%$ respectively. If a randomly selected pair of shoes from the combined output is found to be defective, then what is the chance that the pair was manufactured in the unit U_3?

(A) 30% (B) 15% (C) $\frac{3}{5} \times 100\%$ (D) Cannot be determined from the given data.

3. Consider a paper in the shape of an equilateral triangle ABC with circumcenter O and perimeter 9 units. If we fold the paper in such a way that each of the vertices A, B, C gets identified with O, then the area of the resulting shape in square units is:

(A) $\frac{3\sqrt{3}}{4}$ (B) $\frac{4}{\sqrt{3}}$ (C) $\frac{3\sqrt{3}}{2}$ (D) $3\sqrt{3}$.

4. Let P be a regular twelve-sided polygon. The number of right-angled triangles formed by the vertices of P is

(A) 60 (B) 120 (C) 160 (D) 220.

5. If the n terms $a_1, a_2, \ldots, a_n$ are in arithmetic progression with increment r, then the difference between the mean of their squares and the square of their mean is

(A) $\frac{r^2((n-1)^2-1)}{12}$ (B) $\frac{r^2}{12}$ (C) $\frac{r^2(n^2-1)}{12}$ (D) $\frac{n^2-1}{12}$.

6. A father wants to distribute a certain sum of money between his daughter and son in such a way that if both of them invest their shares in the scheme that offers compound interest at $\frac{25}{3}\%$ per annum, for t and $t+2$ years respectively, then the two shares grow to become equal. If

$-\alpha - 3 \leq 8$

the son's share was rupees 4320, then the total money distributed by the father was

$-5 \quad (6+2) \quad (-5, 3)$

≤ 8

(A) rupees 9360 (B) rupees 9390 (C) rupees 16,590 (D) rupees 16,640.

7. Let α denote a real number. The range of values of $|\alpha - 4|$ such that $|\alpha - 1| + |\alpha + 3| \leq 8$ is

3-4

(A) $(0, 7)$ (B) $(1, 8)$ (C) $[1, 9]$ (D) $[2, 5]$.

8. For each natural number k, choose a complex number z_k with $|z_k| = 1$ and denote by a_k the area of the triangle formed by $z_k, iz_k, z_k + iz_k$. Then, which of the following is true for the series below?

$$\sum_{k=1}^{\infty} (a_k)^k$$

α

(A) It converges only if every z_k lies in the same quadrant. (B) It always diverges. (C) It always converges. (D) none of the above.

9. The function $y = e^{kx}$ satisfies

$$\left(\frac{d^2y}{dx^2} + \frac{dy}{dx}\right)\left(\frac{dy}{dx} - y\right) = y\frac{dy}{dx}$$

for

(A) exactly one value of k. (B) two distinct values of k. (C) three distinct values of k. (D) infinitely many values of k.

10. For a real number θ, consider the following simultaneous equations:

$$(\cos\theta)x - (\sin\theta)y = 1$$

$$(\sin\theta)x + (\cos\theta)y = 2.$$

The number of solutions of these equations in x and y is

(A) 0 (B) 1 (C) infinite for some values of θ (D) finite only when $\theta = \dfrac{m\pi}{n}$ for integers m, and $n \neq 0$.

11. In the range $0 \leq x \leq 2\pi$, the equation $\cos(\sin x) = \frac{1}{2}$ has

(A) 0 solutions. (B) 2 solutions. (C) 4 solutions. (D) infinitely many solutions.

12. A particle is allowed to move in the XY-plane by choosing any one of the two jumps:
1. move two units to right and one unit up, i.e., $(a, b) \mapsto (a+2, b+1)$
or
2. move two units up and one unit to right, i.e., $(a, b) \mapsto (a+1, b+2)$.
Let $P = (30, 63)$ and $Q = (100, 100)$. If the particle starts at the origin, then

(A) P is reachable but not Q.
(B) Q is reachable but not P.
(C) both P and Q are reachable.
(D) neither P nor Q is reachable.

13. For a real polynomial in one variable P, let $Z(P)$ denote the locus of points (x, y) in the plane such that $P(x) + P(y) = 0$. Then,
(A) there exist polynomials Q_1 and Q_2 such that $Z(Q_1)$ is a circle and $Z(Q_2)$ is a parabola.
(B) there does not exist any polynomial Q such that $Z(Q)$ is a circle or a parabola.
(C) there exists a polynomial Q such that $Z(Q)$ is a circle but there does not exist any polynomial P such that $Z(P)$ is a parabola.
(D) there exists a polynomial Q such that $Z(Q)$ is a parabola but there does not exist any polynomial P such that $Z(P)$ is a circle.

14. Let $P(X) = X^4 + a_3X^3 + a_2X^2 + a_1X + a_0$ be a polynomial in X with real coefficients. Assume that

$$P(0) = 1, P(1) = 2, P(2) = 3, \text{ and } P(3) = 4.$$

Then, the value of $P(4)$ is

(A) 5 (B) 24 (C) 29 (D) not determinable from the given data.

15. Let f be a real-valued differentiable function defined on the real line $\mathbb{R}$ such that its derivative f' is zero at exactly two distinct real numbers α and β. Then,

(A) α and β are points of local maxima of the function f.
(B) α and β are points of local minima of the function f.

(C) one must be a point of local maximum and the other must be a point of local minimum of f.

(D) given data is insufficient to conclude about either of them being local extrema points.

16. A school allowed the students of a class to go to swim during the days March 11th to March 15, 2019. The minimum number of students the class should have had that ensures that at least two of them went to swim on the same set of dates is :

(A) 6 (B) 32 (C) 33 (D) 121. 6+

17. Let $a_1 < a_2 < a_3 < a_4$ be positive integers such that $\frac{1}{2}+\frac{1}{4}+\frac{1}{3}+\frac{1}{6}$

$$\sum_{i=1}^{4} \frac{1}{a_i} = \frac{11}{6}.$$

$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}=$

Then, $a_4 - a_2$ equals $\frac{1}{2k}$

$\cancel{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}}$

(A) 11 (B) 10 (C) 9 (D) 8.

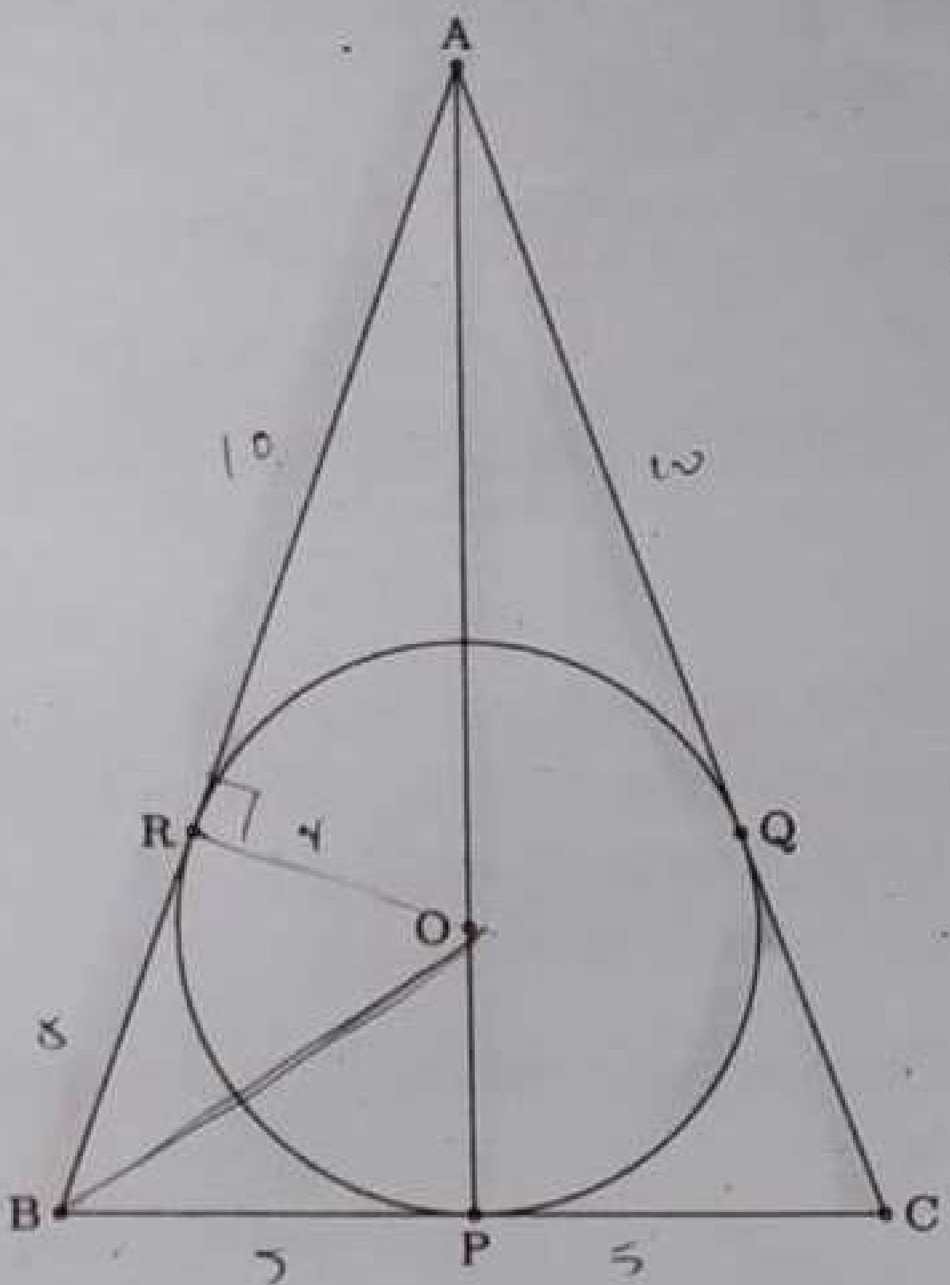
18. Three children and two adults want to cross a river using a rowing boat. The boat can carry no more than a single adult or, in case no adult is in the boat, a maximum of two children. The least number of times the boat needs to cross the river to transport all five people is:

(A) 9 (B) 11 (C) 13 (D) 15.

19. Let M be a 3×3 matrix with all entries being 0 or 1. Then, all possible values for $\det(M)$ are

(A) $0, \pm 1$ (B) $0, \pm 1, \pm 2$ (C) $0, \pm 1, \pm 3$ (D) $0, \pm 1, \pm 2, \pm 3$.

20. In the following picture, ABC is an isosceles triangle with an inscribed circle with center O. Let P be the mid-point of BC. If $AB = AC = 15$ and $BC = 10$, then OP equals:



(A) $\frac{\sqrt{5}}{\sqrt{2}}$ (B) $\frac{5}{\sqrt{2}}$ (C) $2\sqrt{5}$ (D) $5\sqrt{2}$.

$\frac{-2}{-1}$

21. For every real number $x \neq -1$, let $f(x) = \dfrac{x}{x+1}$. Write $f_1(x) = f(x)$ and for $n \geq 2$, $f_n(x) = f(f_{n-1}(x))$. Then,

$$f_1(-2) \cdot f_2(-2) \cdot \cdots \cdot f_n(-2)$$

$f_2(x) = f(f$

must equal

$=$

(A) $\frac{2^n}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}$ (B) 1 (C) $\frac{1}{2}\binom{2n}{n}$ (D) $\binom{2n}{n}$.

22. Let the integers a_i for $0 \leq i \leq 54$ be defined by the equation

$$(1 + X + X^2)^{27} = a_0 + a_1 X + a_2 X^2 + \cdots + a_{54} X^{54}.$$

$f_2(x) =$

Then, $a_0 + a_3 + a_6 + a_9 + \cdots + a_{54}$ equals

$f_3(x) =$

(A) 3^{26} (B) 3^{27} (C) 3^{28} (D) 3^{29}.

23. An examination has 20 questions. For each question the marks that can be obtained are either -1 or 0 or 4. Let S be the set of possible total marks that a student can score in the examination. Then, the number of elements in S is

(A) 93 (B) 94 (C) 95 (D) 96.

$2 \quad \frac{2}{3} \quad \frac{2}{5} \quad \frac{2}{7}$

24. Chords AB and CD of a circle intersect at right angle at the point P. If the lengths of AP, PB, CP, PD are $2, 6, 3, 4$ units respectively, then the radius of the circle is:

(A) 4 (B) $\frac{\sqrt{65}}{2}$ (C) $\frac{\sqrt{66}}{2}$ (D) $\frac{\sqrt{67}}{2}$

25. The locus of points (x, y) in the plane satisfying $\sin^2 x + \sin^2 y = 1$ consists of

(A) A circle that is centered at the origin.
(B) infinitely many circles that are all centered at the origin.
(C) infinitely many lines with slope ± 1.
(D) finitely many lines with slope ± 1.

26. The number of integers $n \geq 10$ such that the product $\binom{n}{10} \cdot \binom{n+1}{10}$ is a perfect square is

$2^a + 2^b = 160$ 160

$2^{mc} + 2^{nc} = 160$ $= 2^5 5$

(A) 0 (B) 1 (C) 2 (D) 3

16 $2^b(2^a+1) = 160$

$2 + 2^{nc} = 144 + 16$

27. Let $a \geq b \geq c \geq 0$ be integers such that $2^a + 2^b - 2^c = 144$. Then, $a + b - c$ equals:

(A) 7 (B) 8 (C) 9 (D) 10. $2^{mc} + 2^{nc} - 2^c = 144$

28. The number of integers n for which the cubic equation $X^3 - X + n = 0$ has 3 distinct integer solutions is:

(A) 0 (B) 1 (C) 2 (D) infinite. $2^c(2^m + 2^n - 1) = 144$

29. The number of real solutions of the equation $x^2 = e^x$ is:

144

$12^2 = 3^2 \cdot 4^2$

(A) 0 (B) 1 (C) 2 (D) 3. $c = 4$

2^4

30. The number of distinct real roots of the equation $x \sin x + \cos x = x^2$ is

(A) 0 (B) 2 (C) 24 (D) none of the above. $2^a + 2^b = 160$

$x \sin x + \cos x = x^2$

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