# Indian National Physics Olympiad – 2011

Roll Number: P 1 1 _ _ _

INPhO – 2011

Date: 30th January 2011

Duration: **Three Hours**

Maximum Marks: **65**

Please fill in all the data below correctly. The contact details provided here would be used for all further correspondence.

Full Name (BLOCK letters) Ms. / Mr.: ____________________

____________________

Male / Female    Date of Birth (dd/mm/yyyy): ____________________

Name of the school / junior college: ____________________

____________________

Class: XI/ XII    Board: ICSE / CBSE / State Board / Other

Address for correspondence (include city and PIN code): ____________________

____________________

____________________PIN Code: ☐☐☐☐☐☐

Telephone (with area code): ____________________Mobile: ____________________

Email address: ____________________

Besides the International Physics Olympiad, do you also want to be considered for the Asian Physics Olympiad? The APhO - 2011 will be held from April 30- May 07 and your presence will be required in Delhi/Israel from April 22 to May 08, 2011.

Yes/No.

*I have read the procedural rules for INO and agree to abide by them.*

Signature

==================================================

***(Do not write below this line)***

==================================================

**MARKS**

| **Que.** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **Total** |
|---|---|---|---|---|---|---|---|---|
| **Marks** | | | | | | | | |

HOMI BHABHA CENTRE FOR SCIENCE EDUCATION
Tata Institute of Fundamental Research
V. N. Purav Marg, Mankhurd, Mumbai, 400 088

**Instructions:**

1. Write your Roll Number on every page of this booklet.
2. Fill out the attached performance card. **Do not detach it from this booklet.**
3. Booklet consists of 26 pages (excluding this sheet) and seven (7) questions.
4. Questions consist of sub-questions. Write your **detailed answer** in the **blank space** provided below the sub-question and **final answer** to the sub-question in the **smaller box** which follows the blank space.
5. Extra sheets are also attached at the end in case you need more space. You may also use these extra sheets for rough work.
6. Computational tools such as calculators, mobiles, pagers, smart watches, slide rules, log tables etc. are **not** allowed.
7. **This entire booklet must be returned.**

**Table of Information**

| | | | |
|---|---|---|---|
| Speed of light in vacuum | $c$ | = | $3.00 \times 10^{8}$ m·s$^{-1}$ |
| Planck's constant | $h$ | = | $6.63 \times 10^{-34}$ J·s |
| Universal constant of Gravitation | $G$ | = | $6.67 \times 10^{-11}$ N·m$^{2}$·kg$^{-2}$ |
| Magnitude of the electron charge | $e$ | = | $1.60 \times 10^{-19}$ C |
| Mass of the electron | $m_e$ | = | $9.11 \times 10^{-31}$ kg |
| Stefan-Boltzmann constant | $\sigma$ | = | $5.67 \times 10^{-8}$ W·m$^{-2}$ · K$^{-4}$ |
| Permittivity constant | $\epsilon_0$ | = | $8.85 \times 10^{-12}$ F·m$^{-1}$ |
| Permeability constant | $\mu_0$ | = | $4\pi \times 10^{-7}$ H·m$^{-1}$ |
| Acceleration due to gravity | $g$ | = | $9.81$ m·s$^{-2}$ |
| Universal Gas Constant | $R$ | = | $8.31$ J· K$^{-1}$·mole$^{-1}$ |

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1. A long wire of radius '$a$' is carrying a direct current $I$. From its surface at point $A$, an electron of charge $-e \quad (e > 0)$ escapes with velocity $v_0$ perpendicular to this surface (see Fig.(1)). Ignore gravity.

**[ 2.5 + 4 + 1.5 = 8 ]**

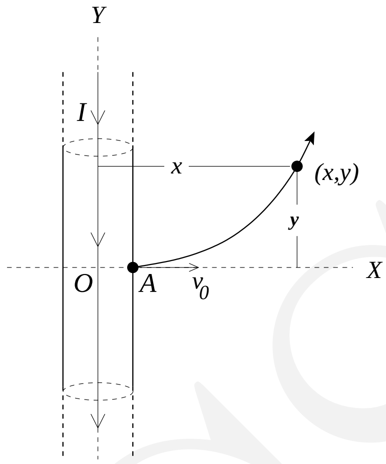



Figure 1:

(a) At $x$ and $y$ the components of the velocity are $v_x$ and $v_y$ respectively. Obtain the components of force $F_x$ and $F_y$ on the electron at any arbitrary point $\{x, y\}$.

| |
|---|
| $F_x =$ |
| $F_y =$ |

(b) Integrate the equation of motion to obtain $v_x$.

$v_x =$

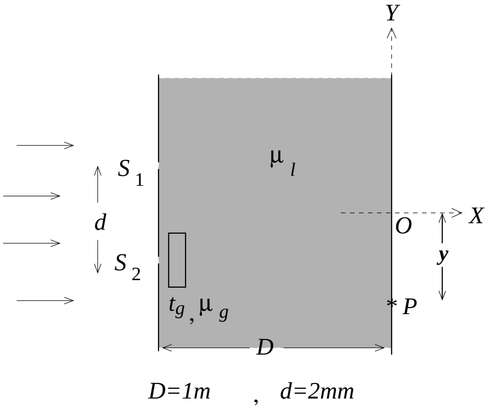
(c) Find the maximum distance $x_{max}$ of electron from the axis of the wire before it turns back.

$x_{max} =$

---

2. In a modified Young's double slit experiment the region between screen and slits is immersed in a liquid whose refractive index varies with time $t$ (in seconds) as $\mu_l = 2.50 - 0.25t$ until it reaches a steady state value 1.25. The distance between the slits and the screen is $D = 1.00$ m and the distance between the slits $S_1$ and $S_2$ is $d = 2.00 \times 10^{-3}$ m. A glass plate of thickness $t_g = 3.60 \times 10^{-5}$ m and refractive index $\mu_g = 1.50$ is introduced in front of one of the slits. Note that the illuminations at $S_1$ and $S_2$ are from coherent sources with zero phase difference.

**[ 2.5 + 2.5 + 1 + 2 + 2 = 10 ]**




(a) Consider the point $P$ on the screen at distance $y$ from $O$ ($S_1O = S_2O$; $OP = y$). Obtain the expression for the optical path difference $\Delta x$ in terms of the refractive indices and the lengths mentioned in the problem.

$\Delta x =$

(b) Now let $P$ denote the central maximum. Obtain the expression for $y$ as a function of time.

$y =$

(c) Obtain the time ($t_m$) when central maximum is at point $O$, equidistant from $S_1$ and $S_2$ i. e. $S_1O = S_2O$.

$t_m =$

(d) What is the speed ($v$) of the central maxima when it is at $O$.

$v =$

(e) If monochromatic light of wavelength 6000 Å is used to illuminate the slits, determine the time interval ($\Delta t$) between two consecutive maxima at $O$ before steady state is reached.

$\Delta t =$

3. A Carnot engine cycle is shown in the Fig. (2). The cycle runs between temperatures $T_H = \alpha T_0$ and $T_L = T_0$ $(\alpha > 1)$. Minimum and maximum volume at state 1 and state 3 are $V_0$ and $nV_0$ respectively. The cycle uses one mole of an ideal gas with $C_P/C_V = \gamma$. Here $C_P$ and $C_V$ are the specific heats at constant pressure and volume respectively. You must express all answers in terms of the given parameters $\{\alpha, n, T_0, V_0, \gamma\}$ and universal gas constant $R$.

**[ 3 + 4 + 1 = 8 ]**

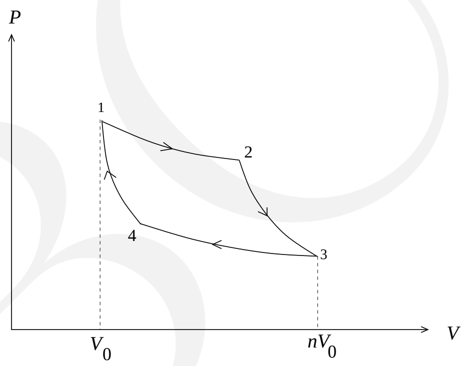



Figure 2:

(a) List $\{P, V, T\}$ of all the four states.

| | | | |
|---|---|---|---|
| $P_1 =$ | $P_2 =$ | $P_3 =$ | $P_4 =$ |
| $V_1 =$ | $V_2 =$ | $V_3 =$ | $V_4 =$ |
| $T_1 =$ | $T_2 =$ | $T_3 =$ | $T_4 =$ |

(b) Calculate the work done by the engine in each process: $W_{12}$, $W_{23}$, $W_{34}$, $W_{41}$.

| |
|---|
| $W_{12} =$ |
| $W_{23} =$ |
| $W_{34} =$ |
| $W_{41} =$ |

(c) Calculate $Q$, the heat absorbed in the cycle.

$Q =$

4. Consider a modification of Coulomb's law by replacing it with the force between two charges $q_1$, $q_2$ separated by $\vec{r}$ given by

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r^2} + \frac{\beta}{r^3}\right]\hat{r}$$

where $\beta$ is a constant. As far as possible express your answers in terms of the standard Bohr radius $a_o = 4\pi\epsilon_0\hbar^2/me^2$ where the symbols have their usual meanings.

**[ 3 + 4 + 1 = 8]**

(a) Obtain the Bohr radius ($r_n$) for this modified law.

$r_n =$

(b) Obtain the expression for the energy ($E_n$) for the $n^{th}$ orbit of this modified law.

$E_n =$

(c) Take $\beta$ to be small ($\beta = 0.1a_0$). Take the binding energy of the standard Bohr hydrogen atom to be 13.60 eV. Calculate the transition energy ($\Delta E$) from $n = 2$ to $n = 1$ for this modified law. For your calculation you may ignore terms of order $\beta^2$ and higher.

$\Delta E =$

5. Consider the motion of electrons in a metal in the presence of electric ($\vec{E}$) and magnetic ($\vec{B}$) fields. Due to collisions there arises a "retarding" force on the electron which is modeled by $m\vec{v}/\tau$ where $m$ is the electron mass, $\vec{v}$ its velocity and $\tau$ a typical collision time. Take the magnitude of the electron charge to be $e$ ( note $e$ is positive). Ignore gravity.

**[ 1 + 2 + 2.5 + 3 + 2 + 1.5 = 12]**

(a) State the equation of motion of the electron.

(b) Consider the case $\vec{E} = 0$ and $\tau \to \infty$. Obtain the expression for the angular cyclotron frequency $\omega_c$ and its numerical value for the case $B = 5.70$ T.

| |
|---|
| $\omega_c =$ |
| Value of $\omega_c =$ |

(c) Consider the case of $\vec{B} = 0$ and $\vec{E} = E\hat{i}$. If $n$ is the number of free (valence) electrons per unit volume, obtain the expression for the conductivity $\sigma_0$ of the sample. Obtain also the numerical value for the conductivity of Cu given that $n = 8.45 \times 10^{28}$ m$^{-3}$ and $\tau = 2.48 \times 10^{-14}$ s. We assume steady state i.e. the acceleration dies down and terminal speed is attained.

| $\sigma_0 =$ |
|---|

| Value of $\sigma_0 =$ |
|---|

(d) Consider the case $\vec{E} = E_y\hat{j} + E_z\hat{k}$ ($E_x = 0$) and $\vec{B} = B\hat{k}$. Assume steady state and relate $\{j_x, j_y, j_z\}$ to $\{E_x E_y, E_z\}$. Here $j$'s are the current densities (current per unit area).

$$j_x = \sigma_{xy}E_y + \sigma_{xz}E_z$$
$$j_y = \sigma_{yy}E_y + \sigma_{yz}E_z$$
$$j_z = \sigma_{zy}E_y + \sigma_{zz}E_z$$

where $\sigma_{ij}$'s are to be written in terms of $\sigma_0$, $\omega_c$ and $\tau$.

| | |
|---|---|
| $\sigma_{xy} =$ | $\sigma_{xz} =$ |
| $\sigma_{yy} =$ | $\sigma_{yz} =$ |
| $\sigma_{zy} =$ | $\sigma_{zz} =$ |

(e) Sketch $j_x$ ($y$-axis) versus $B$ ($x$-axis) .

(f) Taking Cu as an example, for what value of the magnetic field will $j_x$ be a maximum?

| $B =$ |
|---|

6. Two blocks, say $B$ and $C$, each of mass $m$ are connected by a light spring of force constant $k$ and natural length $L$. The whole system is resting on a frictionless table such that $x_B = 0$ and $x_C = L$, where $x_B$ and $x_C$ are the coordinates of the blocks $B$ and $C$ respectively. Another block (named $A$) of mass $M$, which is travelling at speed $V_0$ collides head-on with the block $B$ at an instant $t = 0$ (see Fig. (3)).

**[ 3 + 2 + 2 + 3 = 10]**

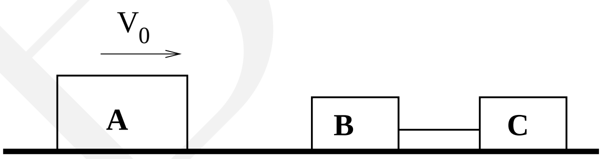



Figure 3:

(a) Obtain the velocities of blocks $A$, $B$ and $C$ just after the collision at $t = 0$? Express the velocities in terms of $V_0$ and $\gamma = m/M$. Assume that the collision is elastic.

| |
|---|
| $V_A =$ |
| $V_B =$ |
| $V_C =$ |

(b) Draw free body diagrams for the blocks $B$ and $C$ after the collison and write down the equation of motion.

Free body diagram:

Equation of motion:

(c) For $t > 0$ the positions of the blocks are given by

$$x_B = \alpha t + \beta \sin(\omega t) \tag{1}$$
$$x_C = L + \alpha t - \beta \sin(\omega t) \tag{2}$$

Find $\omega$ in terms of $m$ and $k$. Express $\alpha$ and $\beta$ in terms of $V_0, \gamma$ and $\omega$. (For this part, ignore the motion of block A.)

| |
|---|
| $\omega =$ |
| $\alpha =$ |
| $\beta =$ |

(d) Obtain the condition on $\gamma$ such that the block $A$ will collide with the block $B$ again at some time $t > 0$?

Condition on $\gamma$:

7. **The Cubic Potential**: Consider a particle of mass $m$ moving in one dimension under the influence of potential energy

$$u(x) = \frac{m\omega^2 x^2}{2} - \delta x - \frac{\alpha x^3}{3}$$

Here $\omega$, $\delta$ and $\alpha$ are real and positive.

**[ 6 + 3 = 9]**

(a) Sketch typical plots of $u(x)$ and identify extrema if any.

(b) Consider the case where (in appropriate units) we have $m = 1$, $\omega = \sqrt{2}$, $\alpha = 1$ and $\delta = 1/2$. Sketch the potential energy $u(x)$. If the total energy of the particle moving in this one-dimensional potential is $E = 0$ (in same units), discuss the motion of the particle in terms of allowed regions, boundedness and periodicity.

***** ***END OF THE QUESTION PAPER*** *****

# Extra Sheet

# Extra Sheet

# Extra Sheet

# Extra Sheet

# Extra Sheet