

KEY

Section 1: Algebra

- 1.1 6
- 1.2 7
- 1.3 $3\mathbb{Z}$
- 1.4 b
- 1.5 3
- 1.6 0
- 1.7 $(-1)^n$
- 1.8 $0, \mathbf{x}^T \mathbf{y}$
- 1.9 a,b,c
- 1.10 $n^2 - 2$

Section 2: Analysis

- 2.1 $\sqrt{a^2 + 1}$
- 2.2 a,c
- 2.3 a,b,c
- 2.4 $\frac{1}{2}$
- 2.5 a,b,c
- 2.6 a,c
- 2.7 a
- 2.8 $\pm \left(\sqrt{\frac{\sqrt{5}+1}{2}} + i\sqrt{\frac{\sqrt{5}-1}{2}} \right)$
- 2.9 $4\pi i$
- 2.10 None

Section 3: Topology

- 3.1 b,c
- 3.2 b,c
- 3.3 a,b,c
- 3.4 b,c
- 3.5 b
- 3.6 c
- 3.7 a,c
- 3.8 a,b
- 3.9 a,c
- 3.10 a

Section 4: Applied Mathematics

- 4.1 $\frac{s}{s^2+4}$
- 4.2 $\beta_N = N\alpha_N$
- 4.3 $\frac{\log 5}{\log 2}$ hours
- 4.4 $\frac{dv}{dt} = g - cv^2$
- 4.5 $v(t) = \sqrt{\frac{g}{c}} \frac{1 - e^{-2\sqrt{gc}t}}{1 + e^{-2\sqrt{gc}t}}$

- 4.6 $\{(x, 0) : x \in \mathbb{R}\}$

4.7 Elliptic

4.8 2

4.9 Min. : $3w_1 - 3w_2 - 4w_3 + 5w_4$
such that

$$\begin{aligned} w_1 - w_2 - 2w_3 + w_4 &\geq 2 \\ 2w_1 - 2w_2 - w_3 + w_4 &\geq 3 \\ w_i &\geq 0, \quad 1 \leq i \leq 4. \end{aligned}$$

4.10 $x_{n+1} = \frac{x_n^2 + a}{2x_n}$

Section 5: Miscellaneous

- 5.1 $2te^{-t^4}$
- 5.2 $\frac{e^3 - e}{2}$
- 5.3 $\log 3$
- 5.4 $(\frac{2}{3}, 0)$
- 5.5 $\frac{5}{2} \sin \frac{2\pi}{5}$
- 5.6 a,b,c
- 5.7 $\binom{r+b-1}{r}$
- 5.8 (a) $\frac{1}{3}$; (b) $\frac{1}{2}$
- 5.9 1
- 5.10 2,3,5,7,13